Fast, Analytical Method for Structured Identification of SISO RC-Ladder-Type Systems

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***Abstract*—Structured system identification often requires solving optimization problems that are not deterministic in computation time and may converge to a local optimum. For the case of systems that can be represented as a SISO "RC-ladder" impedance network, this brief presents a closed-form algorithm that can determine the structured state-space matrices and extract the parameters from an arbitrarily transformed system, such as that produced by subspace system identification. This algorithm relies on a modified version of the Lanczos tridiagonalization process and the solving of a least squares problem with dimension equal to the system order. It is fast, deterministic, and successful for systems of low to moderate order. Practical applications include network synthesis, thermal model identification, and distributed sensing reconstruction, where exact RC parameter values must be identified from data.**

***Index Terms*—Structured identification, RC-ladder, distributed parameter systems, distributed sensing, tridiagonalization, Lanczos process.**

I. Introduction

SYSTEMS of the "RC-ladder" form are commonly encountered in the areas of network synthesis [1], thermal modelling [2], control [3], VLSI propagation delay modelling [4], and distributed systems [5], [6], among others. Identification of RC network parameters at each of the ladder stages is an important problem, which is difficult with general system identification approaches. Conventional approaches yield input-output correct representations from which extracting the parameters is mathematically complex. A deterministic algorithm to identify the parameters of RC-ladder systems from data would allow fast, accurate reconstruction, having applications to the aforementioned fields where RC-ladder models are prevalent.

As an electrical circuit, RC-ladder impedance networks are formed of cascaded 1-port resistor-capacitor (RC) stages, as shown in Figure 1. These systems are characterized by having a Cauer I type continued fraction expansion impedance transfer function [7], which may be formed recursively following (1).

$$Z_i(s) = \begin{cases} \frac{1}{j\omega C_i + \frac{1}{Z_{i-1}(s)}} + R_i & i > 1 \\ \frac{1}{j\omega C_1} + R_1 & i = 1 \end{cases} \tag{1}$$

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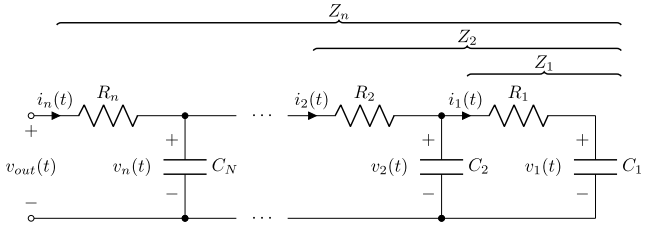

Fig. 1. Model circuit for an n-stage RC-ladder 1-port impedance network.

In the admittance form, the RC-ladder system of interest may be placed in the state-space structure of (2) (order $n = 4$ shown) by taking the capacitor voltages as states. A full discussion on the identifiability of systems of the type (2) is out of the scope of this brief, however identifiability of similar systems is often discussed in the context of network reconstruction [8]. Here, a similar approach to that of [9] is followed, with focus placed on a method for structural identification, assuming identifiability.

Identifying RC-ladder systems from data is important in electrical and thermal modelling, but few methods exist which identify the R_i and C_i parameters directly. RC network identification using deconvolution is able to produce an impedance model and estimate poles and zeros from data [10]. General "black-box" system identification in the state-space domain has become efficient and popular since the introduction of subspace system identification techniques such as the N4SID algorithm [11]. This algorithm and similar ones estimate a state-space model from data that correctly models the input-output behaviour of the system, but which has arbitrary matrix structure and states. Mathematically, they yield a state-space system estimate $\hat{Y}$ which is related to the structured system by an unknown similarity transformation T such that:

$$\mathcal{A}(\vec{\theta}) = T^{-1}\hat{A}T$$
$$\mathcal{C}(\vec{\theta}) = \hat{C}T \tag{3}$$
$$\mathcal{B}(\vec{\theta}) = T^{-1}\hat{B} \tag{4}$$
$$\mathcal{D}(\vec{\theta}) = \hat{D}, \tag{5}$$

where $\vec{\theta}$ is the vector of parameters (i.e., R_i and C_i) that enter arbitrarily in the system matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, and $\mathcal{D}$.

Retrieving the unknown matrix T to transform the unstructured system $\hat{Y}$ into the structured form of (2) is desired so that the state variables and parameters are known and have physical interpretation. General structured system identification is a difficult and well researched problem, which often reduces to a bilinear, non-convex optimization step [12]. Especially for

large order systems, this optimization is highly sensitive to the choice of initial conditions [13] and has no guarantees of convergence to the global optimum. Several methods have been studied to avoid or at least reduce the scale of the non-convex optimization, in order to have deterministic results and short computation time. For example, machine learning techniques may be applied to identify stochastic Markov jump systems [14]. For systems of a specific form (often with the state-space $\mathcal{A}$ matrix having all unknown parameters in a single row or column and an entirely known $\mathcal{C}$ matrix), the full set of parameters and elements of T can be found by solving a null space problem [15]. However, this structure is very limiting and the scale of the problem grows rapidly at $O(n^2)$ with the system order. Various algebraic techniques have been developed to increase the generality of this approach [16], even allowing some complex electrical networks to be parametrically identified [17]. Alternatively, the problem can be cast as a convex optimization of the Hankel matrix subject to a (non-convex) rank constraint, which can then be tackled with an iterative procedure of solving convex semidefinite programs [12].

Interconnected system identification of large scale networks has produced many fruitful techniques in the literature that aim to identify a multiplicity of simple systems while preserving the global structure that interconnects them [18]. For example, this can be done by estimating the hidden states that connect each subsystem from a linear combination of inputs and outputs from those of its local neighbourhood [19]. Applying these to the RC-ladder network seems promising in order to identify each RC filter stage subsystem, from which the R_i and C_i parameters could be easily extracted. Unfortunately, these methods require networks where each local system has external inputs and outputs and is connected only by hidden states. Thus, they are not directly applicable to the SISO RC-ladder case of interest here.

Some structured identification methods have been developed specifically for systems of RC-ladder type (or similar). From the Routh array calculated from the Markov coefficients, a closed-form algorithm exists to yield the Cauer I state-space matrices [7]. This algorithm is a recursive procedure on the impulse response. More recently, a parameterization was proposed that frames the problem as a semidefinite program [9]. This method is applicable to more general RC networks (as opposed to SISO ladder structures) but can be solved in a convex fashion only for autonomous systems (i.e., without input; state-space B matrix absent). However, autonomous systems of this form have limited practicality outside of network reconstruction, and solving the non-convex problem still requires that all elements of $\mathcal{B}$ are known and that global optimization be used, which quickly becomes intractable.

In this brief, we propose an optimization-free, algebraic method for obtaining the structured state-space matrices for RC-ladder systems of low to moderate order. To the best of our knowledge, the Routh array procedure from [7] is the only other algebraic method of parametric identification for these systems. It is nevertheless prone to recursive accumulation of error and does not directly yield the parameters, rather the structured matrices only. We will now describe a method to transform a black-box identified RC-ladder system into the form of (2) and extract the R_i and C_i parameters from it, solving the parametric identification problem in a efficient manner. In Section III, we demonstrate our algorithm with a Monte Carlo simulation, comparing it to [7], [12], and to general grey-box optimization-based system identification.

II. Structured Identification Procedure

A. Tridiagonalization of the State Transition Matrix

Assuming an arbitrarily transformed system $\hat{Y}$ exists (e.g., from subspace methods), the identification algorithm centres around finding a unique similarity transformation T that can tridiagonalize the $\hat{A}$ matrix and transform the remaining state-space matrices into the structure of (2). We will use the following remarks about the special form of $\mathcal{A}$:

(i) The state transition matrix $\mathcal{A}$ has tridiagonal form: $\mathcal{A}_{ij} = 0 \quad \forall \quad \{(i,j) \mid (i-j)^2 > 1\}$.

(ii) The diagonal elements of $\mathcal{A}$ are all strictly negative: $\mathcal{A}_{ii} < 0, \quad i = 1, \ldots, n$.

(iii) The co-diagonal elements of $\mathcal{A}$ are all strictly positive: $\mathcal{A}_{ij} > 0 \quad \forall \quad \{(i,j) \mid (i-j)^2 = 1\}$.

(iv) $\mathcal{A}$ has only distinct, real eigenvalues [20, Corollary 3.1.1].

(v) All but the last row of $\mathcal{A}$ sum to zero: $\sum_{j=1}^{n} \mathcal{A}_{ij} = 0 \quad \forall \quad \{i = 1, \ldots, n-1\}$.

Remarks (i)–(iii) define $\mathcal{A}$ as a "fixed-free" Jacobi structured matrix, implying Remark (iv). The specific tridiagonal structure of $\mathcal{A}$ presents a starting point to find the similarity

$$\mathcal{Y}: \overbrace{\frac{d}{dt}\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix}}^{\dot{\vec{x}}} = \overbrace{\begin{bmatrix} -\frac{1}{R_1C_1} & \frac{1}{R_1C_1} & 0 & 0 \\ \frac{1}{R_1C_2} & -\frac{1}{C_2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & \frac{1}{R_2C_2} & 0 \\ 0 & \frac{1}{R_2C_3} & -\frac{1}{C_3}\left(\frac{1}{R_2}+\frac{1}{R_3}\right) & \frac{1}{R_3C_3} \\ 0 & 0 & \frac{1}{R_3C_4} & -\frac{1}{C_4}\left(\frac{1}{R_3}+\frac{1}{R_4}\right) \end{bmatrix}}^{\mathcal{A}} \overbrace{\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix}}^{\vec{x}} + \overbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{R_4C_4} \end{bmatrix}}^{\mathcal{B}} v_{out}(t)$$

$$i_N(t) = \underbrace{\begin{bmatrix} 0 & 0 & 0 & -\frac{1}{R_4} \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ v_4(t) \end{bmatrix} + \underbrace{\frac{1}{R_4}}_{\mathcal{D}} v_{out}(t) \tag{2}$$

transformation T that transforms the identified system $\hat{Y}$ into its structured representation.

B. Existence and Uniqueness of the Tridiagonal Form

An important result regarding tridiagonalization states that there exists a similarity transformation that tridiagonalizes every real square matrix [21]. The Lanczos process [22] is a commonly used method to find such a transformation. Given a square $n \times n$ matrix $\hat{A}$ and two starting vectors $\vec{p}_n$ and $\vec{q}_n$, an invertible matrix Q exists such that $Q^{-1}\hat{A}Q = A_T$, where A_T is a tridiagonal matrix similar to $\hat{A}$ and Q has last column $\vec{q}_n$. This tridiagonalization is unique up to a diagonal scaling, but for the moment, we can choose A_T as having the particular scaling such that its upper codiagonal is composed of only ones. Furthermore, an invertible matrix P with last column $\vec{p}_n$ exists that satisfies (6).

$$P^T\hat{A}Q = \Omega A_T = A_S \tag{6}$$

where Ω is a particular diagonal scaling matrix that symmetrizes A_T to produce symmetric tridiagonal matrix A_S. The matrices Q, P, A_T, Ω, and A_S are all uniquely determined to within the diagonal scaling by $\vec{q}_n$ and $\vec{p}_n$ [23, Th. 2.1]. For now, we will continue with the Lanczos tridiagonalization and leave the diagonal scaling to be addressed in Section II-D.

C. Lanczos Starting Vectors

The tridiagonalizing similarity transformation matrices Q and P may be defined in terms of their column vectors:

$$Q = \begin{bmatrix} | & & | \\ \vec{q}_1 & \cdots & \vec{q}_n \\ | & & | \end{bmatrix} \tag{7}$$

$$P = Q^{-T}\Omega = \begin{bmatrix} | & & | \\ \vec{p}_1 & \cdots & \vec{p}_n \\ | & & | \end{bmatrix}, \tag{8}$$

The vectors $\vec{q}_n$ and $\vec{p}_n$ necessary to define the transformation can be expressed in terms of the unstructured system. Substituting (7) into (4) with $T = Q$, only the last column of Q remains. We solve for $\vec{q}_n$ in terms of the RC parameters:

$$\vec{q}_n = \hat{D}^{-1}C_n\hat{B} = R_nC_n\hat{B}. \tag{9}$$

Likewise, substituting (8) into (3) with $T^{-1} = P^T$ gives:

$$\vec{p}_n = -\hat{D}^{-1}\hat{C}^T = -R_n\hat{C}^T. \tag{10}$$

The parameter R_n in (9) and (10) is known from (5). Together, (9) and (10) fulfill the conditions for tridiagonalization using the Lanczos method. Provided the Lanczos process is successful, the matrix A_T shown in (11) is produced, which is tridiagonal but still differs from the desired structured $\mathcal{A}$ by the unknown diagonal scaling:

$$A_T = Q^{-1}\hat{A}Q \tag{11}$$

$$A_T = \begin{bmatrix} a_{1,1} & 1 & & & \\ a_{2,1} & a_{2,2} & 1 & & \\ & a_{3,2} & a_{3,3} & \ddots & \\ & & \ddots & \ddots & 1 \\ & & & a_{n,n-1} & a_{n,n} \end{bmatrix}. \tag{12}$$

The parameter C_n from (9) is not known, but is a scalar multiplier to the entire $\vec{q}_n$ vector and Q matrix. This parameter can therefore be factored out of Q and cancels out in (11), so that A_T will still be reconstructed correctly. Using P^T to transform $\hat{B}$ and $\hat{C}$ according to (13) and (14) retains the unknown C_n parameter only in matrix B_T, where it can later be extracted from the structured form following the remarks in Section II-E.

$$B_T = P^T\hat{B} \tag{13}$$

$$C_T = \hat{C}P^{-T} \tag{14}$$

D. Restoring the Diagonal Scaling

The nonsymmetric tridiagonal A_T achieved by the Lanczos process has additonal degrees of freedom in the unknown diagonal scaling. Specifying either the $k = +1$ or $k = -1$ diagonal fixes the other [24, Sec. 9.4.3]. Also, the elementwise product between these diagonals is invariant under diagonal scaling [23, Sec. 2.1]. We may apply a similarity transformation to (12) by a diagonal matrix S, shown in (15), that scales the codiagonal products by scaling factors $\beta_1, \beta_2, \ldots, \beta_{n-1}$ without modifying the main diagonal elements:

$$S = \operatorname{diag}\left(\left[\textstyle\prod_{i=1}^{n-1}\beta_i \quad \prod_{i=2}^{n-1}\beta_i \quad \cdots \quad \beta_{n-1} \quad 1\right]\right) \tag{15}$$

$$S^{-1}A_TS = \begin{bmatrix} a_{1,1} & \beta_1 & & & \\ \frac{a_{2,1}}{\beta_1} & a_{2,2} & \beta_2 & & \\ & \frac{a_{3,2}}{\beta_2} & a_{3,3} & \ddots & \\ & & \ddots & \ddots & \beta_{n-1} \\ & & & \frac{a_{n,n-1}}{\beta_{n-1}} & a_{n,n} \end{bmatrix} \tag{16}$$

Using Remark (v), a small least-squares problem may be formulated to solve for the elements of S. Multiplying (16) through by the scaling factors to eliminate reciprocals of β sets up a system of equations in redefined variables $s_i = \prod_i^{n-1}\beta_i$. The last row of the correctly scaled (16) does not sum to zero, but its sum is equal to the last element of the matrix B_T, known from (13).

Solving (17) determines the entries of S, which may then be used to reconstruct $\hat{\mathcal{A}}(\vec{\theta})$ with the correct diagonal scaling. The presence of 1 as the lower-right element of S means that this similarity transformation does not modify B_T or C_T.

$$A_T \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{n-1} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{R_nC_n} \end{bmatrix} \tag{17}$$

E. Complete Algorithm[1]

To complete the procedure, a method for extracting the RC parameters from the structured system is required. Assuming the system has been placed in the form of (2), the Doyle augmented matrix $\hat{\mathcal{Y}} = \left[\begin{array}{c|c} \hat{\mathcal{A}}(\vec{\theta}) & \hat{\mathcal{B}}(\vec{\theta}) \\ \hline \hat{\mathcal{C}}(\vec{\theta}) & \hat{\mathcal{D}}(\vec{\theta}) \end{array}\right] = \left[\begin{array}{c|c} S^{-1}A_TS & B_T \\ \hline C_T & \hat{D} \end{array}\right]$

[1]Code is provided in our repository: https://gitlab.ethz.ch/BMHT/publications/rcladder-id.

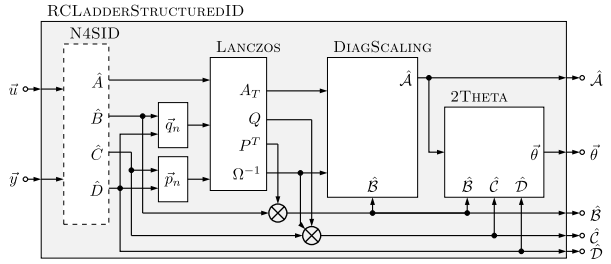

Fig. 2. Signal flow diagram for the full procedure of Algorithm 1.

is formed by concatenation. An LU decomposition of $\hat{\mathcal{Y}}$ (e.g., efficiently using the well-known Crout method) produces the matrices:

$$L = \begin{bmatrix} 1 & & & & \\ -\frac{C_1}{C_2} & 1 & & & \\ & \ddots & \ddots & & \\ & & -\frac{C_{n-1}}{C_n} & 1 & \\ & & & -C_n & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} -\frac{1}{R_1 C_1} & \frac{1}{R_1 C_1} & & & \\ & \ddots & \ddots & & \\ & & & -\frac{1}{R_n C_n} & \frac{1}{R_n C_n} \\ & & & & \frac{2}{R_n} \end{bmatrix}.$$

The convention of having all ones on the diagonal of L ensures a unique decomposition. Taking the $k = -1$ diagonal of L as $\vec{l}$ and the $k = +1$ diagonal of U as $\vec{u}$, the C_i and R_i parameters can then be isolated using (18) and (19), respectively.

$$C_i = \prod_{j=n}^{i} -\vec{l}_j \tag{18}$$

$$R_i = \frac{1}{\vec{u}_i} C_i \tag{19}$$

Figure 2 and the Algorithm 1 pseudocode present the full procedure to identify the R_i and C_i parameters from input-output data by first using standard subspace identification methods to estimate a transformed system. Then, the procedures described in this section exactly transform the arbitrary system into the desired structured form.

Algorithm 1 Parameter Identification From RC-Ladder Model

procedure RCLadderStructuredID($\vec{u}, \vec{y}$)
  $\hat{A}, \hat{B}, \hat{C}, \hat{D} = \text{N4SID}(\vec{u}, \vec{y})$ ▷ Black-box ID [11]
  $\vec{q}_n = 1/\hat{D} \cdot \hat{B}$
  $\vec{p}_n = -1/\hat{D} \cdot \hat{C}$
  $A_T, Q, P^T, \Omega = \text{Lanczos}(\hat{A}, \vec{p}_n, \vec{q}_n)$ ▷ See [23]
  $A_{est}, S = \text{RCLadderDiagScaling}(\Omega^{-1} A_T, P^T \hat{B})$
  $\vec{\theta} = \text{RCLadder2Theta}(A_{est}, P^T \hat{B}, \hat{C} Q \Omega^{-1}, \hat{D})$
  return $\vec{\theta}$
end procedure
procedure RCLadderDiagScaling(A, B)
  $\vec{b} = A[:, \texttt{end}] - B$ ▷ Scaled rows of A sum to $\vec{b}$
  $A \leftarrow A[:, 1\texttt{:end-1}]$
  $\vec{s} = (A^T A)^{-1} A^T \vec{b}$ ▷ Solve least-squares problem
  $S = \text{diag}([\vec{s}; 1])$
  $A \leftarrow S^{-1} A S$
  return A, S
end procedure
procedure RCLadder2Theta(A, B, C, D)
  $Y = [[A, B]; [C, D]]$
  $U, L = \text{Crout}(Y)$ ▷ LU decomp. of tridiag. matrix
  $\vec{c} = -\text{diag}(L, -1)$
  for $i = n - 1,\ i > 0,\ i \leftarrow i - 1$ **do**
    $\vec{c}[i] \leftarrow \vec{c}[i] \cdot \vec{c}[i + 1]$
  end for
  $\vec{r} = \text{diag}(U, 1)$
  for $i = 1,\ i \leq n,\ i \leftarrow i + 1$ **do**
    $\vec{r}[i] \leftarrow \vec{c}[i] / \vec{r}[i]$
  end for
  $\vec{\theta} = [\vec{r}; \vec{c}]$
  return $\vec{\theta}$
end procedure

III. Examples and Discussion

A. Reconstructing Randomly Transformed Systems

Algorithm performance was evaluated by running a Monte Carlo simulation. 1900 "true" state-space models of order 2–20 were created by randomly drawing R_i and C_i values from a uniform distribution in the intervals [10 kΩ, 100 kΩ] and [10 pF, 100 pF], respectively. The systems were then transformed by a random $\mathbb{R}^{n \times n}$ similarity transformation matrix with elements drawn from a $N \sim (0, 1)$ normal distribution. Algorithm 1 was used to reconstruct the "true" systems. Structured identification was considered correct if the 2-norm of the differences between true and reconstructed state-space matrices were all less than 1%. Figure 3 shows the fraction of correct reconstructions out of 100 trials for each system order and the median computation time when run on a PC (Intel Core i9-10920X CPU, 3.5 GHz) for the algorithm described here versus the Routh array method from [7].

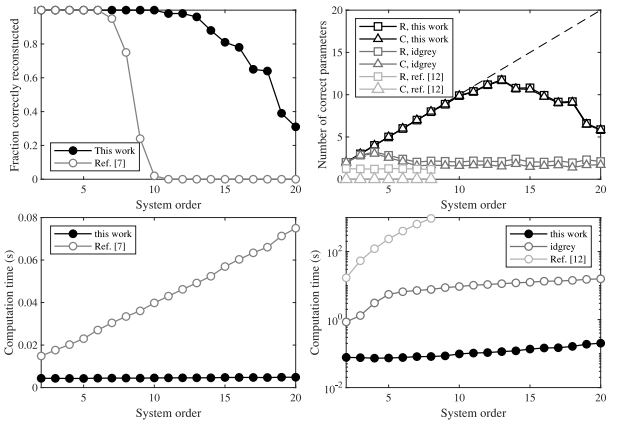

Fig. 3. Left: a comparison of reconstruction accuracy (upper left) and computation time (lower left) between our algorithm and [7] on randomly transformed systems. Right: a comparison of accuracy (upper right) and computation time (lower right, log scale) between our algorithm, [12], and the MATLAB `idgrey` command on identifying the RC parameters from data.

We observe the proposed method results in nearly perfect reconstruction for models of orders 2–11. For higher orders (especially above 15), numerical instability causes breakdown in the Lanczos process. The naïve orthogonal Lanczos process used here has no provisions to detect or correct breakdown. Correction involves detecting the onset of breakdown and continuing with new vectors $\vec{p}_n$ and $\vec{q}_n$. This would unfortunately invalidate (9) and (10). If the Lanczos tridiagonalization succeeds without breakdown, then the algorithm will converge to the unique solution of parameters. For low to moderate order

systems, we see empirically that breakdown is avoided and the algorithm quickly obtains the unique solution. The Routh array method has higher computation time and becomes numerically unstable, even with systems of relatively low order ($n > 6$), because its recursive nature leads to compounding errors.

B. Parametric Identification From Data

The previous example demonstrates the algorithm's performance in restoring the structure to randomly transformed models. In practice, the unstructured model must first come from another technique such as subspace identification. In this example, the random models from Section III-A were simulated to yield a frequency response at 128 logarithmically-spaced points in the range [1 kHz, 10 MHz], without noise. From these frequency-domain data, the N4SID algorithm was used to estimate a continuous time, black-box system. The structured state-space system and parameters were determined from the black-box system using Algorithm 1. Results were compared to the MATLAB `idgrey` command seeded with random initial values, which uses optimization to find the parameters of a grey-box model. We also compared with the general structured identification method from [12]. Each of the $2 \cdot n$ parameters were considered successfully identified if they were within 1% of the true value. Figure 3 shows the results of this comparison for each model order with its respective median computation time. Optimization methods for grey-box identification such as [12] and `idgrey` are versatile but slow and reconstruction accuracy is low for systems with entirely unknown parameters. The method from [12] was not run with systems of order $n > 8$ because of unreasonably long computation times. With random initial values, `idgrey` was unreliable for systems of order 5 or greater and was approximately 250 times slower than even the Routh array method. However, the speed and accuracy of the `idgrey` results would likely greatly improve with a better initial guess.

IV. Conclusion

Although general structured identification methods are powerful and versatile, we present here a faster and more successful approach for the specific case of RC-ladder systems with all unknown parameters. Our method outperforms both the only other known algebraic method and also general optimization-based approaches, while having much lower computation time. This algorithm allows the deterministic, online solving of the structured system identification problem involving small to moderate scale SISO RC-ladder systems of practical interest.

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